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The Importance of Fuzziness in Fuzzy Logic Controllers

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Abstract

The importance of fuzzy number functions on fuzzy logic based controllers are presented. Since the fuzziness of physical systems are represented by fuzzy numbers, it is important to define the fuzzy numbers with such a fuzziness that suits best for the system. Different fuzzy set functions that have become standard functions in literature are used to represent the control actions to be taken. Since the fuzzy functions are different in shapes, this study also compares the usefulness of standardized membership functions in the application of fuzzy logic to control systems. A 25 rule-based fuzzy logic controller is designed and applied to the speed control of a permanent magnet direct current motor. Triangle, trapezoid, sinusoid, bell, gaussian, and Cauchy type standardized membership functions are used to define the fuzziness of the fuzzy variables. The system is simulated using the above mentioned fuzzy membership functions with the same rule base, and under the same operating conditions.

Key Words: Fuzzy logic control, fuzziness, fuzzy membership functions

1. Introduction

The concept of analyzing systems consisting of uncertainty and nonlinearity has gained a new step with the application of fuzzy logic and fuzzy set theory. Especially, the utilization of fuzzy logic in control systems has become very popular. Since a fuzzy logic based controller adjusts the system input to get a desired output by just looking at the output without any mathematical model that is required by classical control systems, it operates in the same way as a human operator does. Therefore, it is possible to get desired control actions for complex, uncertain, and nonlinear systems via fuzzy logic controllers (FLC) without the requirement of their mathematical models and parameter estimation.

A lot of fuzzy logic based control applications can be found in literature [1]. Different fuzzy sets have been used In all these applications to represent the linguistic terms and definitions. The shapes of the fuzzy sets or fuzzy numbers are usually determined using some functions or geometric shapes such as triangular and trapezoid, sine, gaussian, bell, and Cauchy functions. Depending on application type and the system to be controlled, a different fuzzy number or membership function becomes more suitable and gives better performance than the others.

In this study, the membership functions listed above are used in speed control of a permanent magnet direct current motor (PMDCM), and their performances are observed and compared with each others. Since the fuzziness of these membership functions are different, their effects on the PMDCM speed control are also different. Therefore this study is mainly focused on the effects of fuzziness of the membership functions on the control process.

In order to investigate the fuzziness of different membership functions, which may also be called fuzzy numbers, on the control process, a single rule assignment table is constructed and used with each one of the membership functions. For an accurate comparison of the fuzziness of the membership functions, a common base or ground is needed to be used. Since the shapes of the membership functions are different and they all are used to control the same system, the rule decision (assignment) table is chosen as the common ground. Therefore a rule decision table with 25 rules is obtained and used for all the membership functions without altering its structure. A 25 rule decision table is used because five fuzzy numbers namely negative big (NB), negative small (NS), zero (ZZ), positive small (PS), and positive big (PB) are used for each type of the membership functions in order to represent the control actions to be taken.

2. The System to be controlled

The system studied here consists of mainly an FLC, a DC chopper, and a PMDC motor. The general control block diagram of the overall system is given in Figure 1.

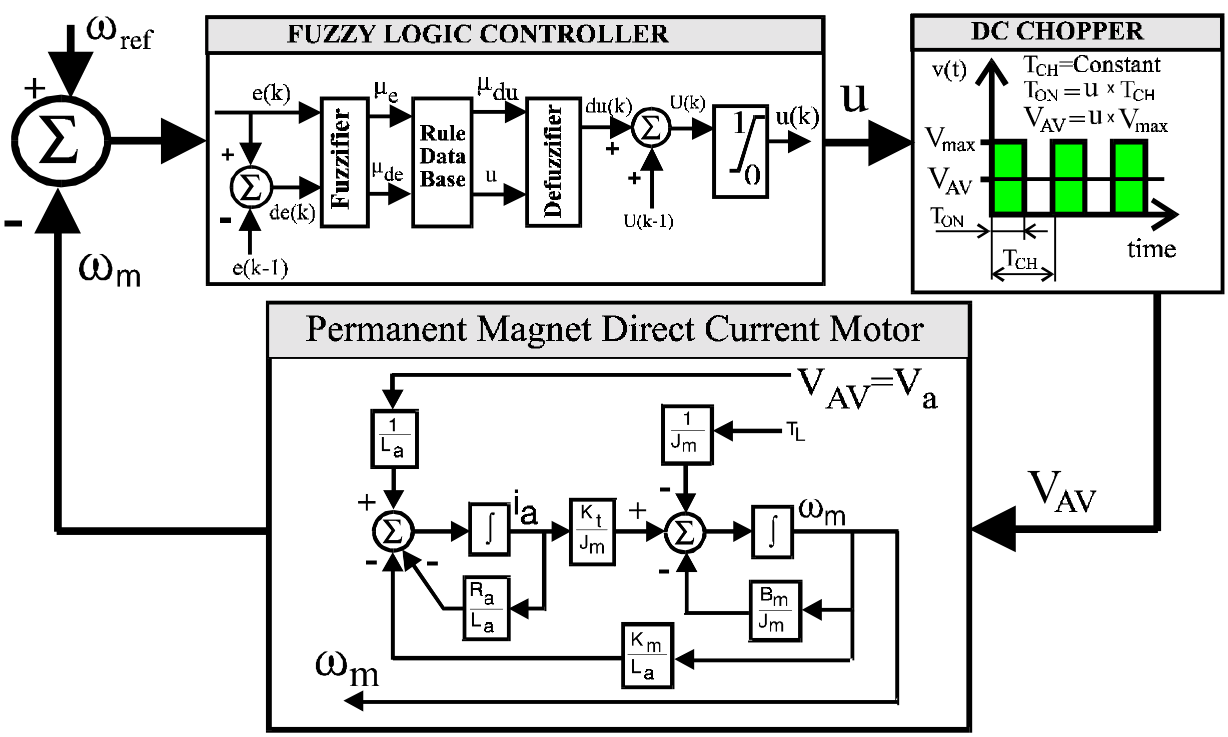


Figure 1. General control block diagram of the system.

Although, it was mentioned above that, the mathematical model of the system to be controlled is not needed in fuzzy logic based control systems, one is needed here since the system is to be simulated. In real time implementation a mathematical model of the systems may not be required, but it is for simulation. Therefore the simulation model of the PMDC motor is represented by its simulation diagram in the related block in Figure 1. The DC chopper is also depicted in detail to show its operational effect on the voltage applied to the motor. As it can be readily understood, the average output voltage, V_{AV}, of the chopper is obtained by integrating the chopped input voltage over the chopping period T_{CH}. This integration yields a linear relationship between the input voltage, V_{max}, and the average output voltage V_{AV} as $V_{AV} = u \times V_{max}$, where the values of the multiplier u is determined by FLC between 0 and 1. As u gets values from 0 to 1, the average output voltage gets values from 0 to V_{max}. The multiplier u is the signal that is used to obtain gating signal, and therefore conducting period T_{ON} of the chopper. Thus, it is important to obtain such a u that enables the input voltage V_a=V_{AV} to be adjusted from 0 Volts to V_{max} Volts.

3 Fuzzy Logic Controller (FLC)

The operation principle of a FL controller is similar to a human operator. It performs the same actions as a human operator does by adjusting the input signal looking at only the system output. A FL based controller consists of three sections namely *fuzzifier, rule base,* and *defuzzifier* as shown in the related block in Figure 1. Two input signals, error and change in error, to the FL controller are converted to fuzzy numbers first in fuzzifier. Then they are used in the rule table to determine the fuzzy number of the output control signal. Finally the resultant fuzzy numbers representing the controller output are converted to the crisp values in defuzzifier..

Five different fuzzy sets representing linguistic fuzzy variables *negative big* (NB), *negative small* (NS), *zero* (ZZ), *positive small* (PS), and *positive big* (PB) are defined using different set functions that have different fuzziness in nature. 7 different fuzzy functions, namely sinusoid, enlarged sinusoid, triangular, trapezoid, gaussian, bell, and Cauchy are used to represent membership functions with different fuzziness. Except the first two, the other five functions used are the functions that have been appeared in literature as almost standard fuzzy membership functions [2-4].

As mentioned before, the rule base consisting the fuzzy logic control rules is chosen as the common ground for 7 different fuzzy membership functions. This rule base, which is also called rule assignment table, is given in Table I. With the each one of the 7 membership function types, the same control output from the table is obtained for the same fuzzy inputs of speed error e and error change de. For example, according to Rule 4, the output du is always NS for all 7 membership functions as long as the inputs e and de are NB and PS, respectively. Similarly, according to Rule 18, the output du is always PS for all different function types as long as e is PS and de is ZZ. The same applies for all of the 25 rules of the rule base.

Rule 4 : ***IF*** *(e is NB)* ***AND*** *(de is PS)* ***THEN*** *(du is NS)*
Rule 18 : ***IF*** *(e is PS)* ***AND*** *(de is ZZ)* ***THEN*** *(du is PS)*

	NB_{de}	NS_{de}	ZZ_{de}	PS_{de}	PB_{de}
NB_e	NB_{du} 1	NB_{du} 2	NS_{du} 3	NS_{du} 4	ZZ_{du} 5
NS_e	NB_{du} 6	NS_{du} 7	NS_{du} 8	ZZ_{du} 9	PS_{du} 10
ZZ_e	NS_{du} 11	NS_{du} 12	ZZ_{du} 13	PS_{du} 14	PS_{du} 15
PS_e	NS_{du} 16	ZZ_{du} 17	PS_{du} 18	PS_{du} 19	PB_{du} 20
PB_e	ZZ_{du} 21	PS_{du} 22	PS_{du} 23	PB_{du} 24	PB_{du} 25

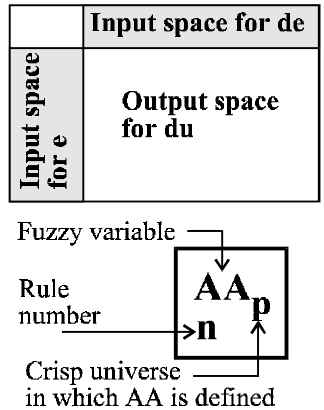


Table 1. Rule assignment table, the rule base.

Implementation of the fuzzifier, rule assignment table, and defuzzifier has been given in previous work [5-9]. Besides, the purpose of this study is to show the effects of fuzziness on FLC rather than to show how an FLC works. Therefore the design and operational procedure of FLC will be skipped.

4. Fuzzy Variables and Fuzziness

Different types of membership functions have been appeared in literature [2-4]. Each one of these functions have different fuzziness in nature depending on parameters used in their definition equations. The functions used here to define the fuzzy membership functions (MF's) negative big (NB), negative small (NS), zero (ZZ), positive small (PS), and positive big (PB) are explained next.

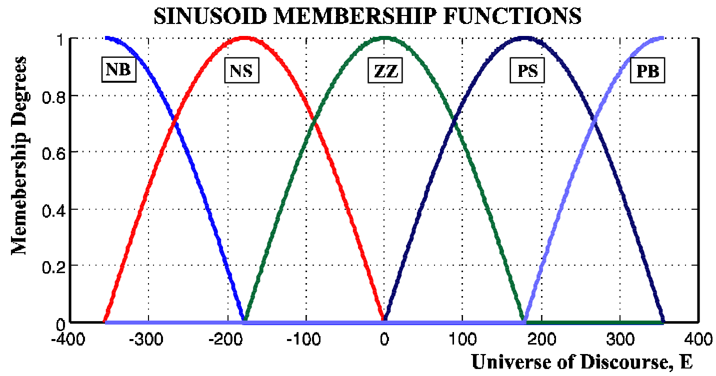
4.1. Sinusoid Membership Functions

This type membership functions are defined in two parts as

$$A_S = \begin{cases} \left|\sin(\omega x + \frac{\pi}{2})\right| & \text{for NB, ZZ, and PB} \\ \left|\sin(\omega x)\right| & \text{for NS and PS} \end{cases} \qquad (1)$$

where the locations of the fuzzy numbers NB, NS, ZZ, PS and PB on the universe of discourse X are in the order of NB<NS<ZZ<PS<PB from negative x to positive x, as shown in Figure 2a. The letter ω in the definition equation is the cycling frequency of the sinusoid functions, and is defined as $\omega=\pi/X_{MAX}$ for 5 fuzzy numbers on one cycle [5].

The fuzziness of sinusoid functions is linear for lower membership degrees, and gets nonlinear around membership degrees close to 1. Therefore the fuzziness of a sinusoid function is similar to that of a triangular for the membership values below 0.6, while it is similar to a gaussian function for membership values above 0.6. Therefore a sinusoid MF may carry the properties of triangular and gaussian functions in some degrees. Sinusoid MF's and resultant speed response of a PMDC motor are given in Figure 2b. The observations and conclusions on results will be given later in "conclusion" section. At this point only the shapes and fuzziness of MF's are focused.



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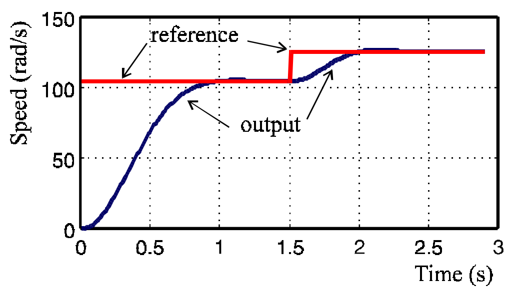

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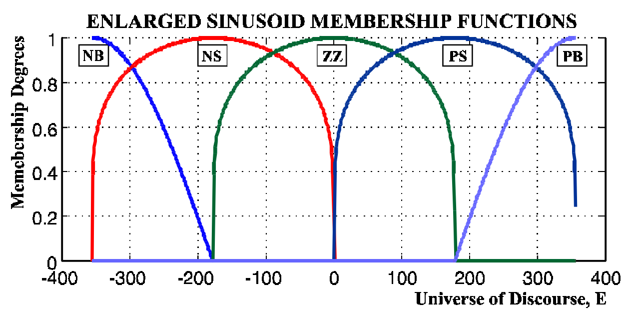
Figure 2. Sinusoidal membership functions and resultant speed responses.

4.2. Enlarged Sinusoid Membership Functions

If sinusoid functions are squared, the MF's shown in Figure 3a are obtained. As shown, the membership values (MV's) below 0.5 are ignored, so that shifting the horizontal axis (the



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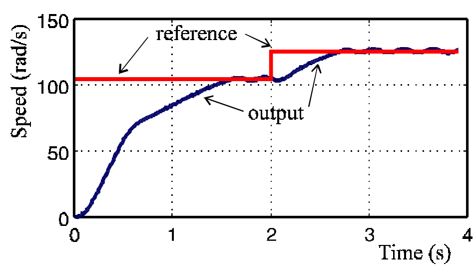

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Figure 3b. Enlarged sinusoidal membership functions and resultant speed response.

axis of the universe of discourse E) from 0 membership value up to 0.5 level does not make any difference. The MF's with MV's greater than 0.5 are highly nonlinear and fuzzy.

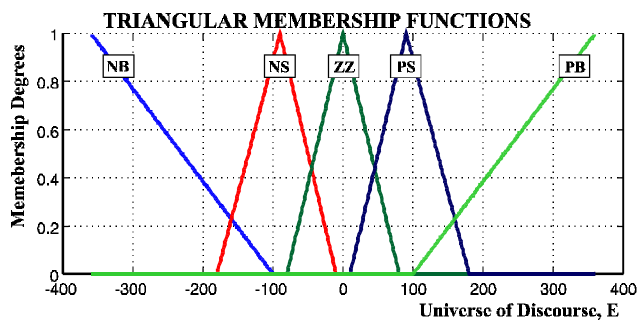
Squaring the sinusoid functions resulted in enlarged type sinusoid functions without lower MV's. Ignoring lower MV's has bad effects on the speed response of the motor. These effects on the speed response will be discussed later.

4.3. Triangular Membership Functions

Triangular membership functions are defined either using parametrized triangular functions or right and left looking line equations. The parametrized equations are defined as

$$A_{\Delta} = \max\left(\min\left(\frac{x - x_L}{x_P - x_L},\ \frac{x_R - x}{x_R - x_P}\right),\ 0\right) \tag{2}$$

Where $x_L < x_P < x_R$, are the crisp x values corresponding to the left lower corner, peak, and right lower corner of triangular membership functions, respectively. A triangular membership function has a linear fuzziness in nature. The distribution of the fuzziness around a certain crisp number with the highest membership value of 1.0, is linear as shown in Figure 4a. Therefore, except the around of sharp peak of a triangle, the other regions of triangular and sinusoid type membership functions show similar responses.



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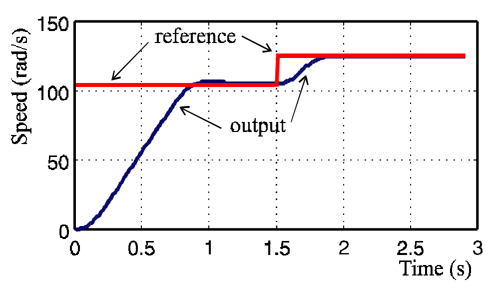

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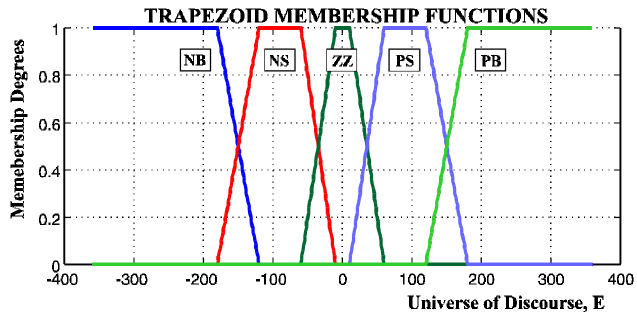
Figure 4. Triangular membership functions and resultant speed response.

4.4 Trapezoid membership functions

Trapezoid membership functions are defined either using parametrized functions or right and left looking line equations as in triangular membership functions. The parametrized equations are defined as

$$A_{T} = \max\left(\min\left(\frac{x - x_L}{x_{P1} - x_L},\ 1,\ \frac{x_R - x}{x_R - x_{P2}}\right),\ 0\right) \tag{3}$$

Where $x_L < x_{P1} < x_{P2} < x_R$, are the crisp x values corresponding to the left lower corner, left peak point, right peak point, and right lower corner of trapezoid membership functions, respectively. If $x_{P1} = x_{P2}$ is taken, than a triangular shape is obtained. As similar to triangular MF's, a trapezoid membership function has a linear fuzziness in nature, as well. The only difference is the crisp band, which has membership degrees of 1. With this type of MF's, the fuzziness near the peak is assumed to be not sensitive to the corresponding crisp values. Since a band with a certain width is assumed to be crisp as shown in Figure 5a, this type of membership functions may result in a steady-state error for control applications. Outside the crisp region, the response of trapezoid MF's, is similar to that of a triangular MF.



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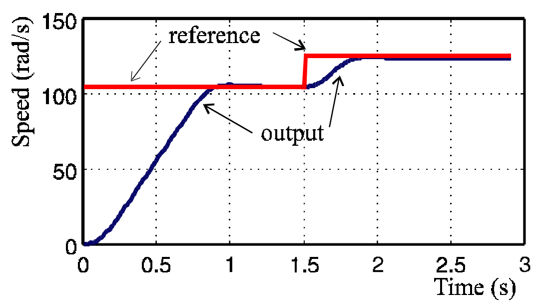

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Figure 5. Trapezoid membership functions and resultant speed response.

4.5. Gaussian membership functions

Parametrized gaussian MF is defined as

$$A_G = e^{-\frac{1}{2}\left(\frac{x - x_P}{w}\right)^2} \tag{4}$$

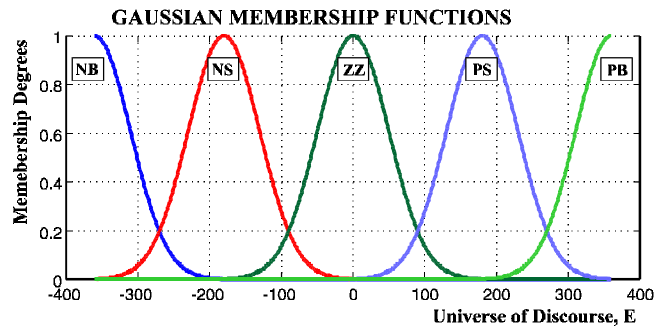
where x_P is the crisp number corresponding to the peak point of the gaussian function. w is a parameter used to define the width, and therefore the fuzziness of gaussian function. The membership functions sown in Figure 6 are obtained with w=50. If a higher value is assigned to w, then the gaussian functions start alternating. For smaller values of w, the width of gaussian functions become narrower resulting in an unsatisfactory speed responses, which require scaling for different reference speed levels.

4.6. Bell membership functions

Parametrized bell MF is defined as

$$A_B = \frac{1}{1 + \left|\frac{x - x_P}{w}\right|^{2m}} \tag{5}$$

where x_P is the crisp number corresponding to the peak point of the bell function. w is a parameter used to define the width, and therefore the fuzziness of bell function. The power m is a parameter used to adjust the width of the crisp peak. The membership functions sown in Figure 7 are obtained with w=100 and m=5. For m=1, Cauchy functions are obtained. If a higher value is assigned to w, then the bell functions become crisp. For smaller values of w, the bell functions turn into Cauchy functions.



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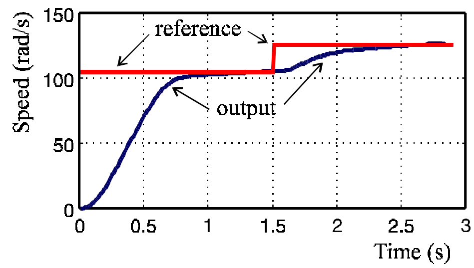

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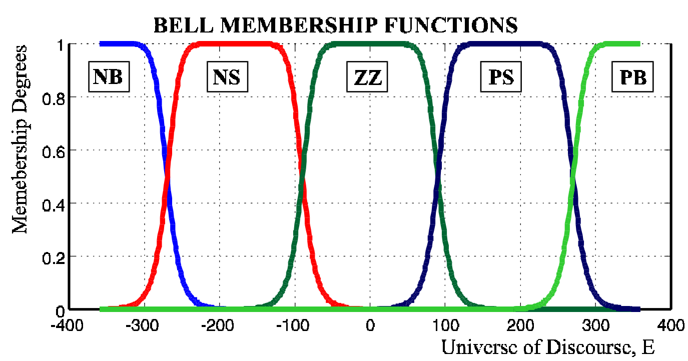
Figure 6. Gaussian membership functions and resultant speed response.



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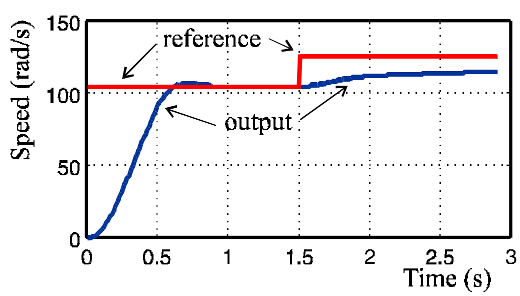

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Figure 7. Bell membership functions and resultant speed response.

4.7. Cauchy membership functions

Parametrized Cauchy MF is defined as

$$A_C = \frac{1}{1+\left(\frac{x-x_P}{w}\right)^{2m}} \tag{6}$$

where x_P is the crisp number corresponding to the peak point of the Cauchy function. w is a parameter used to define the width, and therefore the fuzziness of Cauchy function. The power m is a parameter used to adjust the width of the crisp peak. The membership functions sown in Figure 8 are obtained with w=50 and m=1. Cauchy functions were used in two different ways. As seen in Figure 8a, if the membership values below 0.2 are ignored, the Cauchy functions become look like gaussian functions where a crisp value of E intersects with only one or two subsets at any instant. The resultant speed response with only the membership values over 0.2 is shown in Figure 8b. This response should be compared with the response obtained using gaussian functions. If all of the membership values in the interval [0,1] are considered, a crisp value of E intersects all five fuzzy subsets all the time. The resultant speed response for this latter case is shown in Figure 8c.

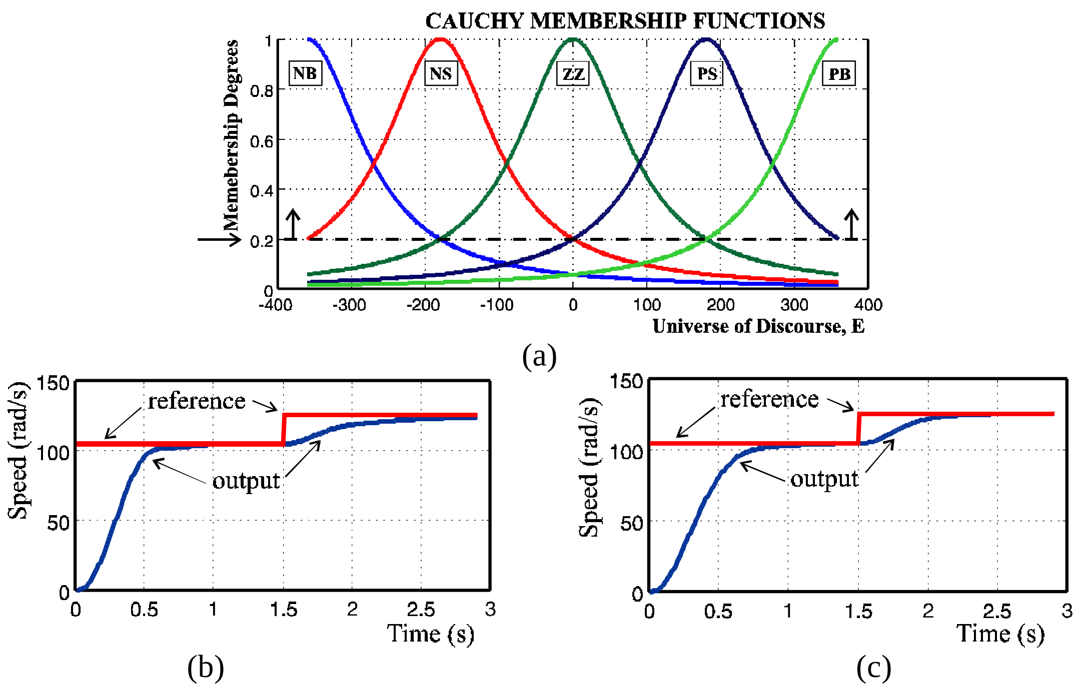

Figure 8. Cauchy membership functions and resultant speed response.

5. Results and Conclusions

Resultant speed responses of the FL controlled PMDC motor scheme described in Figure 1 are given in Figures 2b, 3b, 4b, 5b, 6b, 7b, 8b, and 8c, for different fuzzy subset functions used to represent the fuzzy membership functions NB, NS, ZZ, PS, and PB. The system is simulated in two stages so that transient response of the system is obtained both for startup period and for a step change in reference input speed. While the system is operating at a reference speed of 104.7 rad/sec (1000 rpm), the reference speed is suddenly increased by 20.9 rad/s (200 rpm) to see the response of the controller to a step change in reference. The results with sinusoid, triangular, and trapezoid membership functions show some similarities on settling time. However trapezoid functions give a very small steady-state error, while the other two have very good responses during startup and step change in reference. The similarities in the responses with sinusoid and triangular functions comes from the linearity of the fuzzy membership functions for membership values below about 0.5. As mentioned earlier and shown in Figure 3b, the response with enlarged sinusoid functions is not as good as others. The settling time is longer. Besides, the speed has small fluctuations. The reason of this distorted response is the ignorance of membership values below 0.5 for the enlarged sinusoid functions. The fuzziness of gaussian functions for membership values between 0.2 and 0.8 is almost linear for the parameters selected, as shown in Figure 6a. This linearity is similar to that of sinusoid and triangular functions. Therefore the response with gaussian functions also shows some similarities to the responses with sinusoid and triangular functions. However, the response to the step change is slower than the others due to the larger distances between NB, NS, ZZ, PS, and PB defined by gaussian functions. As seen in Figure 6a, the blank spaces between the gaussian type linguistic fuzzy variables are getting larger as the membership values are increased. Although, the startup transient response with bell functions is acceptable, the response to a step disturbance has a large steady-state error, which is not acceptable. For this type of membership functions, the controller output must be scaled for different reference speed. The response with the bell functions has some similarities to the response with trapezoid functions. This is reasonable because both types of functions give similar subset shapes with the differences at upper and lower corners where they are nonlinear with bell functions. Since the crisp portions of the bell functions with the selected parameters are larger than that of trapezoid functions, the speed response becomes worse. The final conclusion will be about the responses with Cauchy functions, which are given in Figure 8. As mentioned before, Cauchy functions are used in two ways. First the functions are used by ignoring the membership values below 0.2. And secondly, the functions are used as they are. As seen from Figure 8c, the latter case gives better responses. When all the membership values between 0 and 1 are considered, the resultant response is almost the same as that of triangular and sinusoid functions. When the membership values below 0.2 are ignored, the response to a step disturbance become slower.

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